Curvature Analysis of Cardiac Excitation Wavefronts

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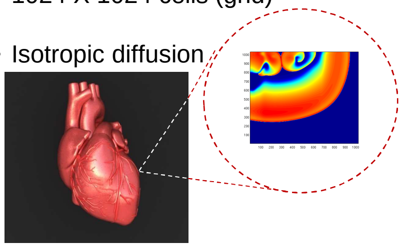
NSF Expeditions in Computing

Which parameter ranges of a DEM reproduce Arrhythmia?

Differential Equation Model (DEM) Simulation

- 1024 X 1024 cells (grid)
- Isotropic diffusion



Strategy: Principled parameter space partitioning [3]

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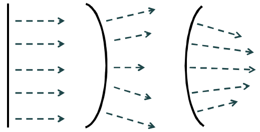
Arrhythmia detection while exploring partitions – Spiral Classification Algorithm (SCA) [1]

$$\theta = \theta_0 - \frac{D}{r}$$

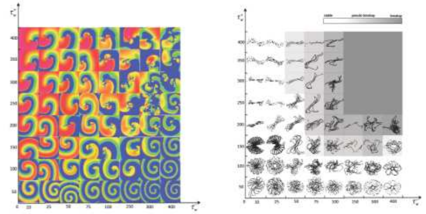
Fibrillation at $r_c = \frac{D}{\theta_0}$



$\theta = \theta_0$ $\theta < \theta_0$ $\theta > \theta_0$

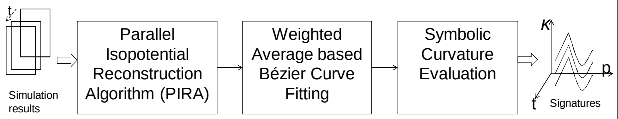
SCA

Distinguishing signatures of arrhythmia based on wave curvature evolution



Parameter analysis of the Minimal model [2]

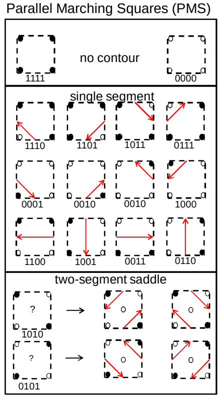
Spiral Classification Algorithm (SCA)



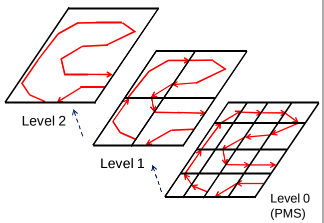
Parallel Isopotential Reconstruction Algo. (PIRA)

- Constructs Isopotentials in each frame of the simulation
- PIRA = PMS + PIE

Parallel Marching Squares (PMS)



Parallel Isoline Extraction (PIE)



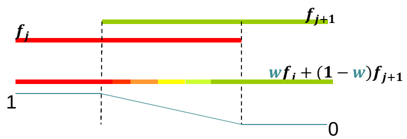
- Implemented on NVIDIA GPUs.
- Up to 444X speedup versus Matlab's *contour.*

Weighted Average based Bézier Curve Fitting

- Fits C2 smooth cubic Bézier curves on to overlapping *strips* of Isopotentials returned by PIRA
- Least squares fitting

$$X_j(t) = (1-t)^3 P_j^0 + 3t(1-t)^2 P_j^1 + 3t^2(1-t) P_j^2 + t^3 P_j^3,$$
$$Y_j(t) = (1-t)^3 Q_j^0 + 3t(1-t)^2 Q_j^1 + 3t^2(1-t) Q_j^2 + t^3 Q_j^3.$$

- Piecewise fits smoothed by weighted average of overlapping adjacent fits f_j, f_{j+1}



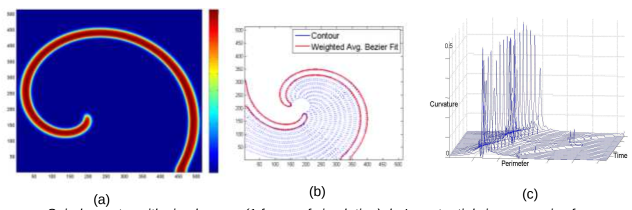
Symbolic Curvature Evaluation

- Symbolic curvature functions constructed using MATLAB's symbolic computation toolbox

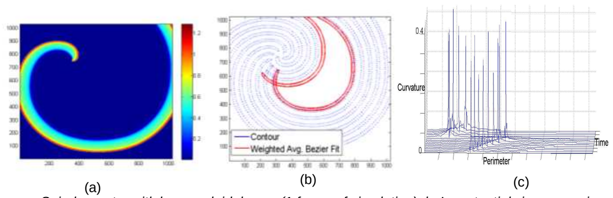
$$\kappa_j(t) = \frac{|r_j'(t) \times r_j''(t)|}{|r_j'(t)|^3} \quad r_j(t) = [X_j(t), Y_j(t)]$$

- Can be evaluated/analyzed at arbitrary spatial resolution

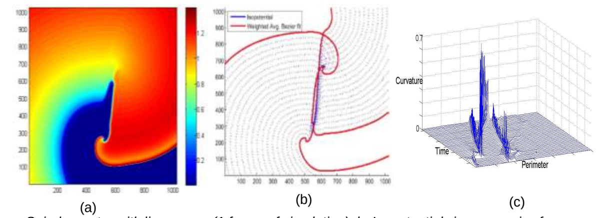
Case Studies



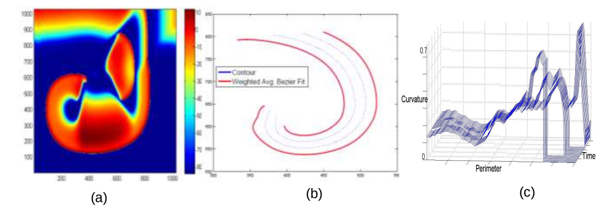
a. Spiral re-entry with circular core (1 frame of simulation). b. Isopotentials in successive frames. c. Curvature evolution.



a. Spiral re-entry with hypocycloidal core (1 frame of simulation). b. Isopotentials in successive frames. c. Curvature evolution.



a. Spiral re-entry with linear core (1 frame of simulation). b. Isopotentials in successive frames. c. Curvature evolution.



a. Spiral wave breakup (1 frame of simulation – post breakup). b. Isopotentials in successive frames till breakup. c. Curvature evolution, with pronounced curvature at point of breakup.

Conclusions and Future Work

- SCA: accurate curvature estimation continuously along the wave
- Curvature evolution based signatures: classify and detect arrhythmia
- Repository of arrhythmia signatures
- GPU-based Bézier curve fitting

REFERENCES

1. A. Murthy et. al., Curvature Analysis of Cardiac Excitation Wavefronts, ACM CMSB 2011. *(invited for extended journal version submission to IEEE-ACM TCBB.)*
2. E. Bartocci et. al. Teaching Cardiac Electrophysiology modeling to undergraduate students: laboratory exercises and GPU programming for study of arrhythmias and spiral wave dynamics, Advances in Physiology Education, 2011.
3. R. Grosu et. al., From Cardiac Cells to Genetic Regulatory Networks, CAV 2011.

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